$$= f(180) + \frac{2}{5}\nabla f(180) + \frac{\frac{2}{5}\left(\frac{2}{5}-1\right)}{\lfloor 2}\nabla^2 f(180)$$

$$+ \frac{\frac{2}{5}\left(\frac{2}{5}-1\right)\left(\frac{2}{5}-2\right)}{\lfloor 3}\nabla^3 f(180) + \frac{\frac{2}{5}\left(\frac{2}{5}-1\right)\left(\frac{2}{5}-2\right)\left(\frac{2}{5}-3\right)}{\lfloor 4}\nabla^4 f(180)$$

$$= 5026 + \frac{2}{5}\times 648 + \frac{2\times -3}{2\times 25}\times 40 + \frac{2\times -3\times -8}{6\times 125}\times -2$$

$$+ \frac{2\times -3\times -8\times -12}{621}\times 4$$

$$= 5280.10 \text{ Ans}$$

Sol^n (3) → Now the following table give the Population of country.

Year	Popl^n $f(x)$ in thousand	$\nabla f(x)$	$\nabla^2 f(x)$	$\nabla^3 f(x)$	$\nabla^4 f(x)$
1891	46	20	−5	2	−3
1901	66	15	−3	−1	
1911	81	12	−4		
1921	93	8			
1931	101				

$x = 1925$

$x_0 = 1891$

$h = 1901 - 1891 = 10$

$u = \frac{x - x_0}{h} = \frac{1925 - 1891}{10} = 3.4$

Then

$$f(x) = f(x_0 + uh) = f(1891) + 3.4\nabla f(1891) + \frac{3.4(3.4-1)}{\lfloor 2}\nabla^2 f(1891)$$

$$+ \frac{3.4(3.4-1)(3.4-2)}{\lfloor 3}\nabla^3 f(1891)$$

$$+ \frac{3.4(3.4-1)(3.4-2)(3.4-3)}{\lfloor 4}\nabla^4 f(1891)$$

$$= 46 + 3.4\times 20 + \frac{3.4\times 2.4}{\lfloor 2}\times -5 + \frac{3.4\times 2.4\times 1.4}{6}\times 2$$

$$+ \frac{3.4\times 2.4\times 1.4\times 0.4}{24}\times -3$$

$$= 46 + 68 - 20.4 + 3808 - 0.5712 = 96.8368$$

(2) Newton's Backward Interpolation formula

This formula is used to explain the backward pops^n of equal intervals & can be written as

$$f(a+nh+uh) = f(a+nh) + u\Delta f(a+nh) + \frac{u(u+1)}{\lfloor 2}\nabla^2 f(a+nh)$$

$$+ \cdots + \frac{u(u+1)\cdots[u+(n-1)h]}{\lfloor n}\nabla^n f(a+nh)$$

Proof → let us consider a func^n $f(x)$ such that $y = f(x)$

where $x = a+nh,\ a+(n-1)h, \ldots a+h$ represent the continuous derivative of a func^n $f(x)$

for this let us consider a func^n $f(x)$ such that

Ques 2 find the value of the area of circle of diameter 82 from the table?

d	80	85	90	95	100
A	5026	5674	6362	7088	7854

5280.16

Ques 3 The $popl^n$ of a country is calculated for the year 1925 from the following data?

(96.84)

Year	1891	1901	1911	1921	1931
$Popl^n$ in lakhs	46	66	81	93	101

Sol^n following table represent the variation in angle obtained by the students

Q.1

angle θ	$f(x)$	$\nabla f(x)$	$\nabla^2 f(x)$	$\nabla^3 f(x)$
45°	0.7071			
		0.0589		
50°	0.7660		-0.0057	
		0.0532		0.0007
55°	0.8192		-0.0064	
		0.0468		
60°	0.8660			

Here $x = 52°$

$x_0 = 45°$

$h = 50° - 45° = 5°$

Then $u = \frac{x - x_0}{h} = \frac{52° - 45°}{5} = \frac{7}{5}$

Then $f(52°) = f(52°) + \frac{7}{5} \times 0.0589 + \frac{\frac{7}{5}\left(\frac{7}{5} - 1\right)}{2} \times -0.0057$

$+ \frac{\frac{7}{5}\left(\frac{7}{5} - 1\right)\left(\frac{7}{5} - 2\right)}{6} \times 0.0007$

$= 0.7071 + 0.08 - \frac{7 \times 2}{50} \times 0.0057 + \frac{7 \times 2 \times -3}{125 \times 6} \times 0.0007$

$= 0.7825$

Ans-2 following table given the area of circle of diameter.

Diameter	$f(x)$	$\nabla f(x)$	$\nabla^2 f(x)$	$\nabla^3 f(x)$	$\nabla^4 f(x)$
80	5026				
		648			
85	5674				
		688	40		
90	6362		38	-2	4
		726		2	
95	7088		40		
		766			
100	7854				

Here $x = 82$

$x_0 = 80$

$h = 85 - 80 = 5$

$u = \frac{x - x_0}{h} = \frac{82 - 80}{5} = \frac{2}{5}$

Then from interpolation formula we know that

$f(x) = f(x_0 + uh) = f(x_0) + u\nabla f(x_0) + \frac{u(u-1)}{\lfloor 2} \nabla^2 f(x_0)$

$+ \frac{u(u-1)(u-2)}{\lfloor 3} \nabla^3 f(x) + \frac{u(u-1)(u-2)(u-3)}{\lfloor 4} \nabla^4 f$

$$f(x) = A_0 + A_1(x-a-nh) + A_2(x-a-nh)(x-a(n-1)h] + A_3(x-a-nh)[x-a-(n-1)h][x-a-(n-2)h] + - - - - + A_n(x-a-nh) - - - (x-a-h)$$

for $x = a + nh$ we get

$$f(a+nh) = A_0 + A_1 \times 0$$

$$\boxed{A_0 = f(a+nh)}$$

llly at $x = a+(n-1)h$ we get -

$$f[a+(n-1)h] = A_0 + A_1[\cancel{a} + (n-1)h - \cancel{a} - nh] + A_2 \times 0$$

$$= A_0 - A_1 h$$

$$A_1 h = A_0 - f[a+(n-1)h]$$

$$A_1 h = f(a+nh) - f[a+(n-1)h]$$

$$\boxed{A_1 = \frac{\nabla f(a+nh)}{h}}$$

also at $x = a+(n-2)h$ we get

$$f[a+(n-2)h] = A_0 + A_1[\cancel{a} + (n-2)h - \cancel{a} - nh] + A_2[a_1 + (n-2)h - \cancel{a} - nh] [\cancel{a} + (n-2)h - \cancel{a} - (n-1)h] + A_3 \times 0$$

$$= A_0 + A_1 \times -2h + A_2 \times -2h - 2h \times (-h)$$

$$f(a+(n-2)h] = A_0 - 2A_1 h + 2h^2 A_2$$

$$2h^2 A_2 = f[a+(n-2)h] + 2\frac{\nabla f(a+nh)}{\cancel{h}} \times \cancel{h}$$

$f(a+nh)$

$$A_2 = \nabla \frac{\ldots}{\lfloor 2\, h^2}$$

$$A_n = \nabla^n \frac{f(a+nh)}{\lfloor n\, h^2}$$

on replacing these values in resultant func^n we get

$$f(x) = f(a+nh) + \frac{\nabla f(a+nh)}{h}(x-a-nh) + \frac{\nabla^2 f(a+nh)}{\lfloor 2\, h^2}(x-a-nh)[x-(n-1)h] + \frac{\nabla^3 f(a+nh)}{\lfloor 3\, h^2}(x-a-nh)[x-(n-1)h][x-(n-2)h] + - -$$

let $u = \frac{x-a-nh}{h} \Rightarrow uh = x-a-nh$

also $u+1 = \frac{x-a-nh}{h} + 1 = \frac{x-a-(n-1)h}{h}$

$$\boxed{f(x) = f(a+nh) + u\nabla f(a+nh) + \frac{u(u+1)\nabla^2 f(a+nh)}{\lfloor 2} + \frac{u(u+1)(u+2)\nabla^3 f(a+nh)}{\lfloor 3} + - - \frac{u(u+1) - [u+(n-1)]\nabla^n f(a+nh)}{\lfloor n}}$$

A